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Relativistic Corrections to the Neutron Electric Dipole Moment in Valence Quark Models

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Abstract

We show that, in valence quark models, the relativistic corrections to the SU(6) relation for the contribution of the electric dipole moments of the quarks to the electric dipole moment of the neutron can be expressed as a multiplicative correction factor. The correction factor is evaluated in light cone wavefunction models, in the bag model, and in relativistic mean-field models, and is found to lie between 1/3 and 1.

We also show that, in these models, there is a linear relation between the correction to the SU(6) value for g_A and that for the electric dipole moment.

I. INTRODUCTION

The neutron electric dipole moment (EDM), D_n, is non-zero only when symmetry under both parity and time-reversal are broken. That P and CP violation have been observed in $K_L \to 2\pi$ makes it plausible that the neutron EDM (*i.e.* a term $D\overline{\psi}\gamma_5\sigma_{\mu\nu}\psi$ in the electromagnetic current) will be observed, although only an upper limit

$$|D_n| < 1.1 \times 10^{-25}\, e \text{ cm} \tag{1}$$

is currently available [1]. Theoretically, the value of D_n is determined by contributions from the electric dipole moments of the quarks (D_q), the colour electric dipole moments of the quarks and gluons, P and T violating q–q interactions and long distance effects [2]. In this paper we concentrate on the contribution from the EDMs of the valence quarks. D_n is generally calculated from the D_q using the non-relativistic SU(6) relation:

$$D_n^{(V)} = \frac{1}{3}(4D_d - D_u) \tag{2}$$

Ellis [3] has emphasised the need for the examination of relativistic corrections to (2). We will refer to the SU(6) value of $D_n^{(V)}$ as D_0, *i.e.*

$$D_0 \equiv \frac{1}{3}(4D_d - D_u)$$

and the relativistic value of $D_n^{(V)}$ as D. The ratio D/D_0 will be referred to as κ_D.

It is reasonable to expect that relativistic effects should be significant. Even one of the key papers establishing the non-relativistic constituent quark model [4] required a constituent quark mass of order 300 MeV and a radius of the hadron of order 0.6 fm $\approx$ 330 MeV^{-1}. Thus $p/m \approx 1$, and relativistic corrections cannot be expected to be negligible.

We can get some idea of what to expect by examining the magnetic dipole moments, where relativistic corrections to the SU(6) relations are known to be important [5–9]. Indeed, in that case the relativistic corrections destroy the agreement between theory and experiment for the octet magnetic moments. We will give the results of an analogous calculation for the electic dipole moments in section II.

Other relativistic models of the valence quark wavefunctions of the hadron are the bag model [10], and the mean-field potential model [11]. The bag model, which does not respect translation invariance, has been criticised as a model of the *magnetic* dipole moments [12]. In the case of the *electric* dipole moment there is no contribution from the γ_μ, so that the corrections from the spurious centre of mass motion in bag models may not be so important. Results from these models are similar and are discussed in section III.

In all of these models we find the same relationship between the relativistic corrections to the EDM and the relativistic corrections to g_A. This relationship is discussed in section IV. Our analysis may be regarded as analogous to the relation between magnetic moments and the axial vector matrix elements discussed by Karl [9].

II. LIGHT CONE WAVEFUNCTION MODELS

In this section, we use the light-front formalism as developed by Berestetskii and Terent'ev [5], in a completely analogous way to their calculation of the nucleon magnetic moments [6]. (The correction to the three-body Melosh transformation [13] is not required for nucleon calculations.) We extend equation (23) of reference [6], which gives the hadron electromagnetic form factors in terms of the electromagnetic interactions of the quarks, to include the electric dipole moment terms:

$$\begin{aligned}
&\left\{F_1^\Lambda \delta_{\lambda\lambda'} + iF_2^\Lambda \left(\boldsymbol{k}\cdot\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}_{\lambda\lambda'}\right) + D^\Lambda \left(\boldsymbol{k}\cdot\boldsymbol{\sigma}_{\lambda\lambda'}\right)\right\}\delta_{\Lambda\Lambda'} \\
&= 3\int d\Gamma \psi^*_{\Lambda'\lambda'}\left\{F_1^{(c)} + iF_2^{(c)}\left(\boldsymbol{k}\cdot\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}^{(c)}\right) + D^{(c)}\left(\boldsymbol{k}\cdot\boldsymbol{\sigma}^{(c)}\right)\right\} e^{-ik\rho^{(c)}}\psi_{\Lambda\lambda}
\end{aligned}$$

Application of the Melosh transformation following the analysis of references [6] and [7] yields the result

$$\kappa_D \equiv \frac{D}{D_0} = 1 - \zeta \tag{3}$$

where ζ is a complicated integral over the quark relative momenta, that depends on the chosen form of the quark wavefunction. The calculation of κ_D is almost identical to the calculation of the contribution of the quark *anomalous magnetic* moments to the anomalous magnetic moment of the nucleon, which is no surprise since

$$\gamma_5 \sigma_{\mu\nu} = \frac{i}{2} \varepsilon_{\mu\nu\rho\lambda} \sigma^{\rho\lambda}$$

One can evaluate the correction factor κ_D above by using some specific choice for the wave-function; for example, with the gaussian form $\Phi \sim \exp(-M_0^2/\alpha^2)$ (where M_0 is the effective mass, which is a function of the momenta of the constituent quarks; see [5]), a constituent quark mass of 300 MeV, and a choice of the free parameter α that correctly reproduces the neutron *magnetic* moment [7], this model gives $\kappa_D = 0.993$, which is a very small variation from the non-relativistic value. For this form of wave function it is possible to demonstrate that κ_D is a monotonically decreasing function of α/m, which approaches the value $\kappa_D = 1/2$ as $\alpha/m \to \infty$ (the extreme relativistic limit).

It is instructive to apply the same technique to the axial vector charged current $A_\mu^{(+)}$. It is well known [4] that the nonrelativistic quark model gives the SU(6) value $g_{A(0)} = 5/3$. The lightcone wavefunction methods applied to $A_\mu^{(+)}$ give

$$\frac{g_A}{g_{A(0)}} = 1 - 2\zeta \tag{4}$$

where ζ is the *same* complicated integral as in (3). (It is amusing to note that the extreme relativistic limit discussed above gives $g_A \to 0$ — we do not have any physical interpretation of this extreme relativistic quenching of g_A, which seems to be a peculiarity of the light cone wavefunction model.)

Eliminating ζ from equations (3) and (4) gives us

$$\kappa_D \equiv \frac{D}{D_0} = \frac{1}{2} \left\{ 1 + \frac{g_A}{g_{A(0)}} \right\} \tag{5}$$

One might then, as an improved estimate, use the *experimental* nucleon axial coupling constant [14]

$$g_A = 1.261 \pm 0.004$$

in equation (5) to obtain

$$\kappa_D \approx 0.878$$

which indicates that the relativistic corrections to the EDM of the hadrons are on the order of 10%.

III. BAG AND MEAN-FIELD POTENTIAL MODELS

For our purposes the bag model may be regarded as equivalent to a potential model, since the essential feature we require is the structure of the single particle relativistic wave function, *viz.*

$$\psi = \begin{pmatrix} f_j(r)\sqrt{\frac{j+m_j}{2j}}\,Y_{j-1/2,m_j-1/2} \\ -f_j(r)\sqrt{\frac{j-m_j}{2j}}\,Y_{j-1/2,m_j+1/2} \\ g_j(r)\sqrt{\frac{j+1-m_j}{2(j+1)}}\,Y_{j+1/2,m_j-1/2} \\ g_j(r)\sqrt{\frac{j+1+m_j}{2(j+1)}}\,Y_{j+1/2,m_j+1/2} \end{pmatrix}$$

where j is the total angular momentum, m_j its projection, $f_j(r)$ and $g_j(r)$ the solutions of the (coupled) radial equations, and we are using the Dirac–Pauli representation of the gamma matrices.

That the bag model determines the radial wavefunctions $f_j(r)$ and $g_j(r)$ by imposing boundary conditions, and that mean-field potential models determine them from an assumed or calculated mean-field potential in the Dirac equation, is not important for our calculation.

Using the P and T violating electromagnetic current

$$J_\mu^{\not P \not T} = D_q \overline{\psi}\gamma_5\sigma_{\mu\nu}\psi$$

and the axial vector current

$$A_\mu^{(+)} = g_A \overline{\psi}\gamma_\mu\gamma_5\psi$$

one readily obtains

$$\frac{D}{D_0} = F_j^2 + \frac{1}{3}G_j^2 \tag{6}$$

and

$$\frac{g_A}{g_{A(0)}} = F_j^2 - \frac{1}{3}G_j^2 \tag{7}$$

where we have defined the quantities

$$F_j^2 \equiv \int_0^\infty r^2 dr \ |f_j(r)|^2$$

and

$$G_j^2 \equiv \int_0^\infty r^2 dr \ |g_j(r)|^2$$

In addition to (6) and (7), normalisation of the wavefunction requires

$$F_j^2 + G_j^2 = 1 \tag{8}$$

Elimination of F_j^2 and G_j^2 from (6) and (7) using (8) gives the relation:

$$\kappa_D = \frac{1}{2}\left\{1 + \frac{g_A}{g_{A(0)}}\right\}$$

just as before.

In this case, using equations (6), (7) and (8), and the fact that F_j^2 and G_j^2 are by definition positive-indefinite, one can also see that the relativistic modification to the valence quark contribution to to g_A and D, *regardless* of the bag boundary conditions or potentials assumed, must be in the range

$$\frac{1}{3} \leq \frac{D}{D_0} \leq 1$$

and

$$-\frac{1}{3} \leq \frac{g_A}{g_{A(0)}} \leq 1$$

respectively.

IV. DISCUSSION

The most significant result of our calculations is that in all the models considered, we find that the relativistic wavefunction corrections to D and g_A are related by

$$\kappa_D \equiv \frac{D}{D_0} = \frac{1}{2}\left\{1 + \frac{g_A}{g_{A(0)}}\right\} \tag{9}$$

We emphasise that all of the models we have considered are independent quark models, which ignore interactions between the quarks, and which ignore QCD except through the "mean field" seen by the quarks. Many contributions to D (*e.g.* quark and gluon electric dipole moments, P and T violating quark–quark interactions, long distance effects), and to g_A (*e.g.* gluonic contributions, anomalous contributions, sea quark effects, long distance effects) have been omitted from our considerations, so that strictly speaking equation (9) applies only to the valence quark contributions to both D and g_A.

However, equation (9) does answer Ellis's call for an estimate of the relativistic corrections to the valence quark contribution to D_n. It shows that the relativistic effects alter the valence quark contributions to D by about 10% — and in the most extreme and unrealistic models by no more than a factor of 3. Given that the level of uncertainty in the estimates of D_q can be as much as a factor of 10, as can the uncertainties in the other effects which can contribute to D [2], we must regard the relativistic corrections considered here as well controlled.

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